Thermodynamics

(a) An ideal gas expands at constant pressure P from volume V_1 to volume V_2.

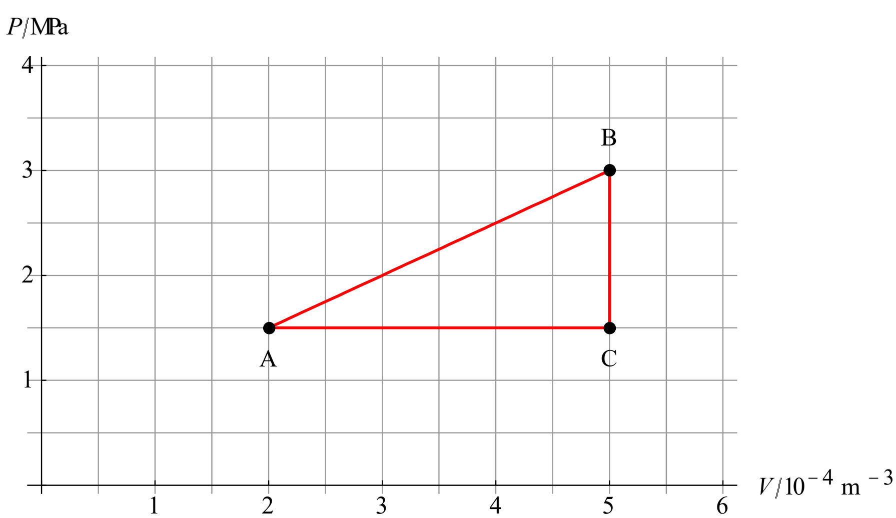
(i) Show that the change in the internal energy of the gas ΔU is given by $\Delta U = \frac{3}{2}P\Delta V$ where $\Delta V = V_2 - V_1$.

(ii) Hence deduce that the thermal energy Q that must be provided is given by $Q = \frac{5}{2}P\Delta V$.

(iii) Thermal energy 1.2 kJ is provided to an ideal gas at constant pressure 4.5 MPa. The initial volume of the gas is 2.4×10^{-4} m^3. Calculate the final volume of the gas.

(iv) Calculate the percentage change in the temperature of the gas.

(b) The P-V diagram shows a cycle ABCA performed by 0.150 moles of an ideal gas.



(i) Calculate the temperature of the gas at A, B and C.

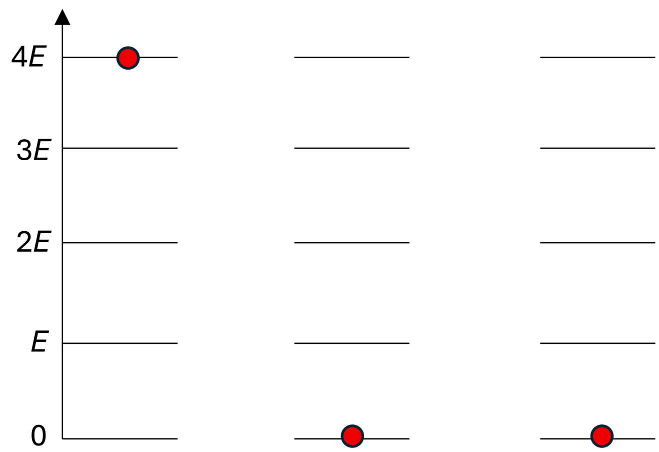
(ii) Determine the thermal energy supplied to the gas during the expansion $A \to B$.

(iii) Estimate the efficiency of the cycle.

(iv) State the point during the cycle where the entropy of the gas is maximum.

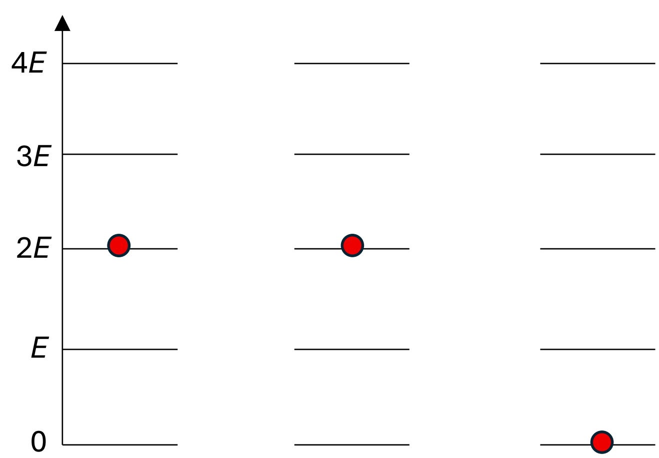
(c) A system consists of three identical particles. Each particle can exist in energy levels quantized in units of E. The total energy of the three particles is $4E$.

(i) One arrangement (Type 1) of total energy $4E$ is shown.



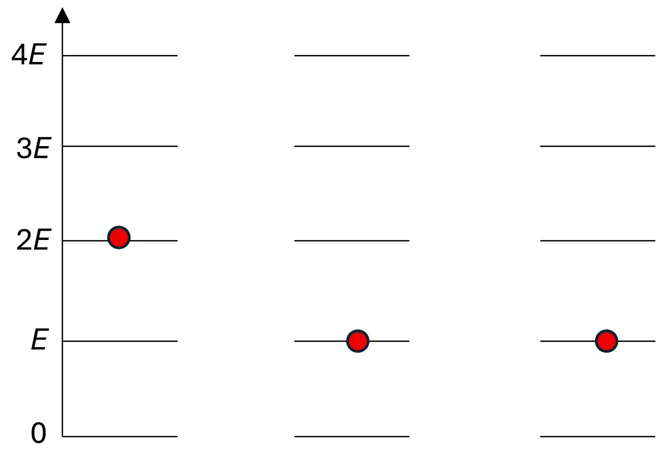
How many such configurations are there?

(ii) Another arrangement (Type 2) is shown:



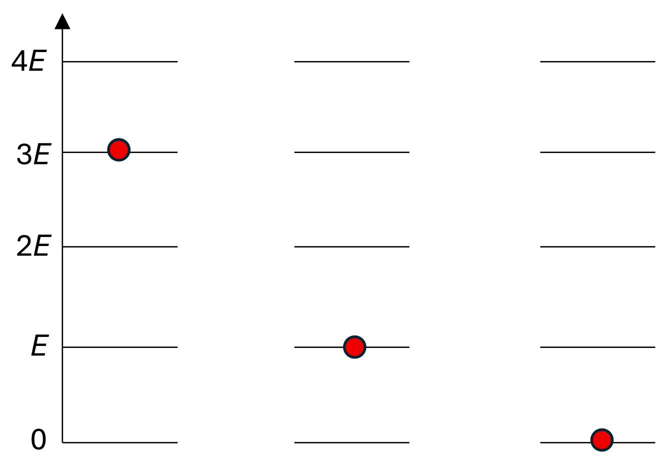
How many such configurations are there?

(iii) Yet another arrangement (Type 3) is:



How many such configurations are there?

(iv) Finally we have (Type 4):



How many such configurations are there?

(v) Hence confirm that the total number of microstates of total energy $4E$ is 15.

(vi) A microstate is chosen at random. In which type of microstate (1, 2, 3, or 4) is the system most likely to be be in?

Answers

(a)

(i)

$$\Delta U = \frac{3}{2}Rn\Delta T$$
$$= \frac{3}{2}Rn(T_2 - T_1)$$
$$= \frac{3}{2}RnT_2 - \frac{3}{2}RnT_1$$
$$= \frac{3}{2}PV_2 - \frac{3}{2}PV_1$$
$$= \frac{3}{2}P\Delta V$$

(ii)

$$Q = \Delta U + P\Delta V$$
$$= \frac{3}{2}P\Delta V + P\Delta V$$
$$= \frac{5}{2}P\Delta V$$

(iii) $1.2\times 10^3 = \frac{5}{2}\times 4.5\times 10^6 \times (V_2 - 2.4\times 10^{-4})$. This gives

$V_2 = 3.47\times 10^{-4} \simeq 3.5\times 10^{-4}$ m^3.

(iv)

$$T_2 - T_1 = \frac{PV_2}{Rn} - \frac{PV_1}{Rn}$$
$$\frac{T_2 - T_1}{T_1} = \frac{\frac{PV_2 - PV_1}{Rn}}{\frac{PV_1}{Rn}}$$
$$= \frac{V_2 - V_1}{V_1}$$
$$= \frac{3.47 - 2.4}{2.4} = 0.446$$

So about 45%.

(b)

(i) $T_A = \frac{P_A V_A}{Rn} = \frac{1.50\times 10^6 \times 2.00\times 10^{-4}}{8.31\times 0.150} = 240.7 \approx 241\,\text{K}$

$T = \frac{PV}{Rn} \Rightarrow T_B = \frac{P_B V_B}{P_A V_A}T_A = \frac{5.0\times 3.0}{1.5\times 2.0}\times 240.7 = 1230\,\text{K}$; $T_C = 1230\times \frac{1.50}{3.00} = 615\,\text{K}$.

(ii) $Q = \Delta U + W$.

$\Delta U = \frac{3}{2}Rn\Delta T = \frac{3}{2}\times 8.31\times 0.150\times(1230.5-240.7) = 1851\text{ J}$.

The work done is the area under the graph for $A \to B$ and equals

$W = (\frac{1.50\times10^{6}+3.00\times10^{6}}{2})\times 3.00\times10^{-4} = 675\text{ J}$. Hence

$Q = 1851 + 675 = 2.53\times10^{3}\text{ J}$

(iii) The net work is the area of the loop i.e. $W = (\frac{1.50\times10^{6}\times3.00\times10^{-4}}{2}) = 225\text{ J}$.

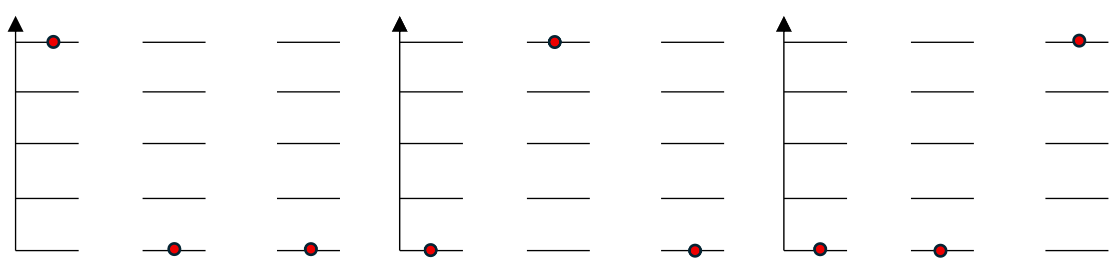
The input thermal energy is that found in (ii) and so

$\eta = \frac{\text{net work}}{\text{input thermal energy}} = \frac{225}{2.53\times10^{3}} = 0.089$ or 8.9%.

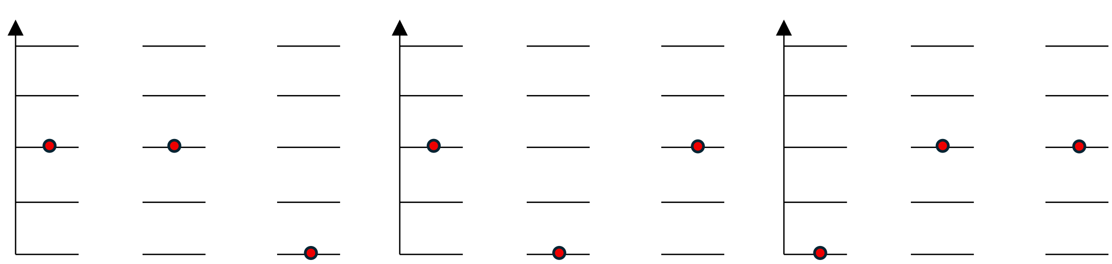
(iv) The entropy of the gas increases from $A \to B$ since thermal energy is provided to the gas. It decreases from $B \to C$ and $C \to A$ since energy is being removed from the gas. (The entropy returns to its original value at A.) Hence the maximum entropy is at B.

(c)

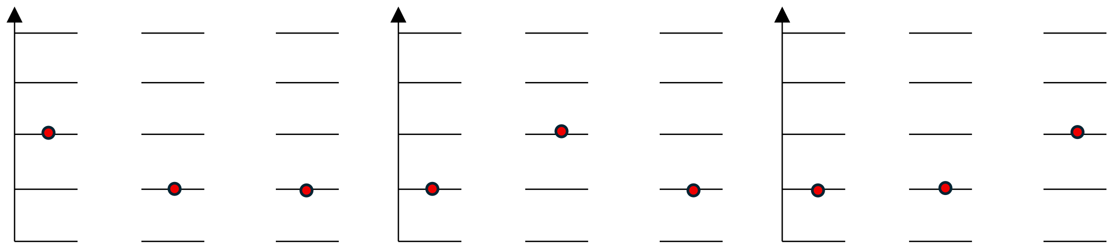
(i) There are 3: there are 3 ways to choose the particle in the state 4*E* with no choice for the others.



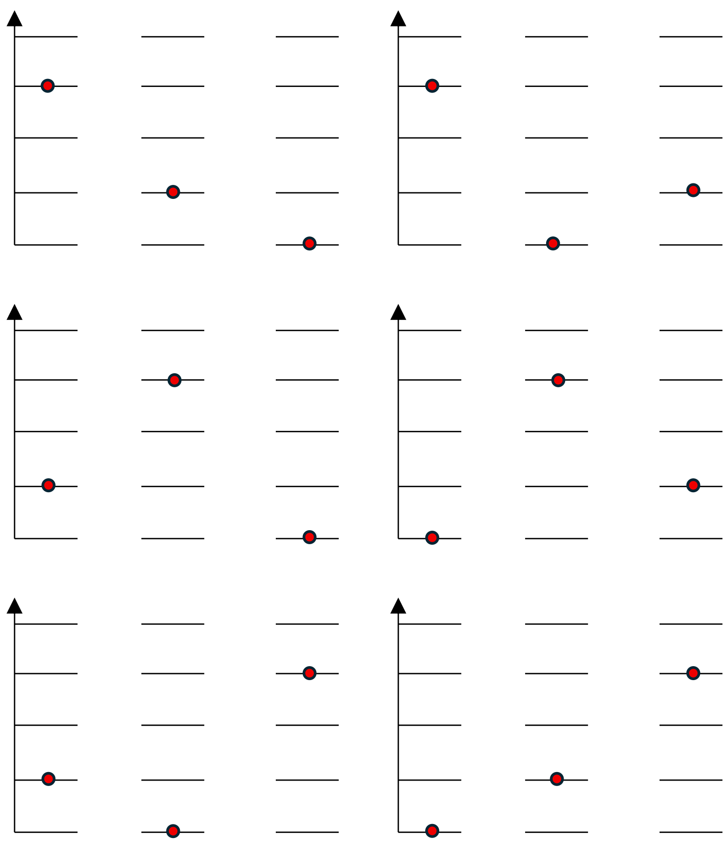
(ii) There are 3: there are 3 ways to choose the particle in the state 0 with no choice for the others.



(iii) There are 3: there are 3 ways to choose the particle in the state $2E$ with no choice for the others.



(iv) There are 6: there are 3 ways to choose the particle in the state $2E$ and 2 choices for the particle in the state $1E$ with no choice for the third giving a total of $3\times 2 = 6$.



(v) 3+3+3+6=15

(vi) Type 4 because there are more microstates of this type. They are also the more disordered with larger variation in how the energy of $4E$ is distributed among the 3 particles (see the Teacher Note *Another view of entropy*).